

# Employing MPI Collectives for Timing Analysis on Embedded Multi-Cores

**Martin Frieb**, Alexander Stegmeier,  
Jörg Mische, Theo Ungerer

Department of Computer Science  
University of Augsburg

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Björn Lisper, WCET 2012:

"Towards Parallel Programming Models for Predictability"

- Shared memory does not scale  
⇒ Replace it with distributed memory
- Replace bus with Network-on-Chip (NoC)
- Learn from Parallel Programming Models
- e.g. Bulk Synchronous Programming (BSP):  
Execute program in *supersteps*:
  1. Local computation
  2. Global communication
  3. Barrier

Similar programming model comes with MPI programs

- At a *collective operation*, all (or a group of) cores work together
- local computation, followed by communication  
⇒ implicit barrier
- One core for coordination and distribution (master), others for computation (slave)
- Examples:
  - Barrier
  - Broadcast
  - Global sum



Background

Timing Analysis of MPI Collective Operations

Case Study: Timing Analysis of the CG Benchmark

Summary and Outlook



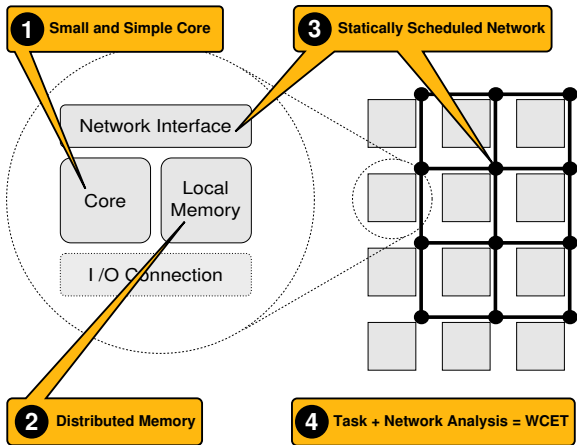
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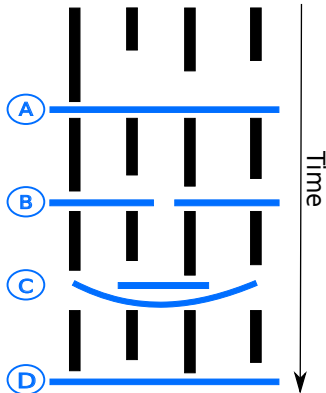
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# Underlying Architecture



[Metzlaff et al.: A Real-Time Capable Many-Core Model, RTSS-WiP 2012]

# Structure of a MPI program



Same sequential code on all cores

(A) Barrier after initialization

(B) Data exchange

(C) Data exchange

(D) Global operation



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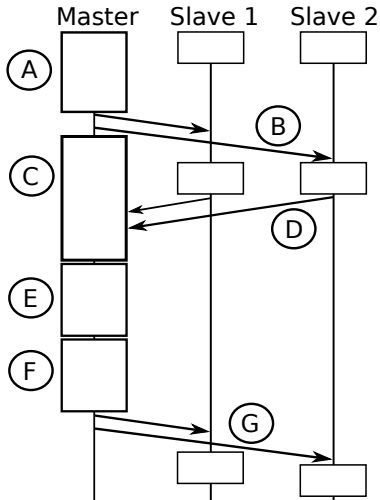




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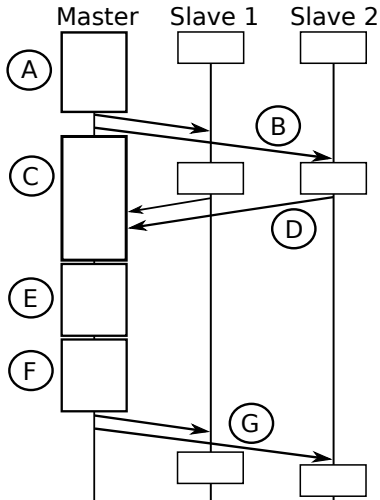
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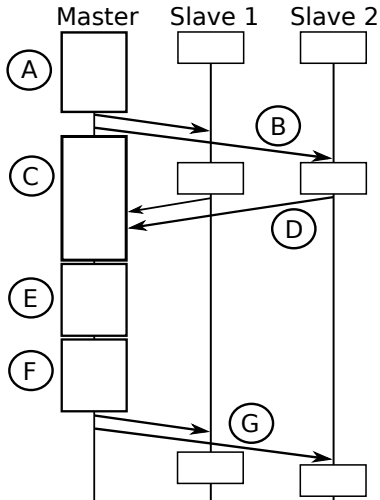
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(A) Initialization

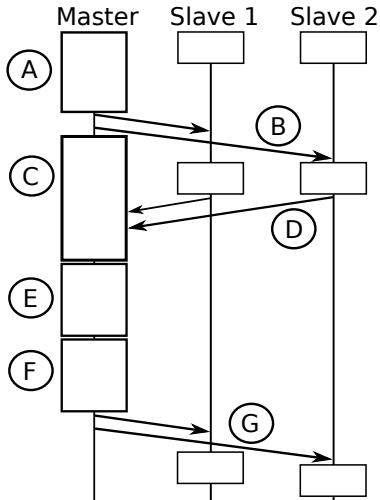
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- Global reduction operation
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- (A) Initialization
- (B) Acknowledgement

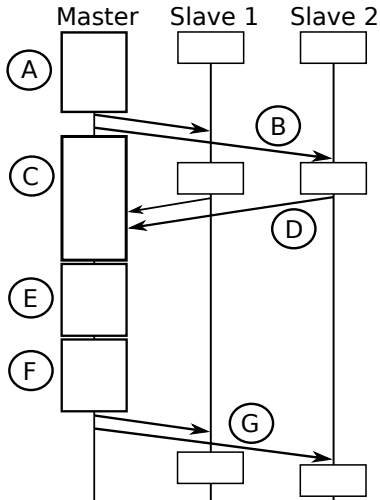
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- (D) Send values

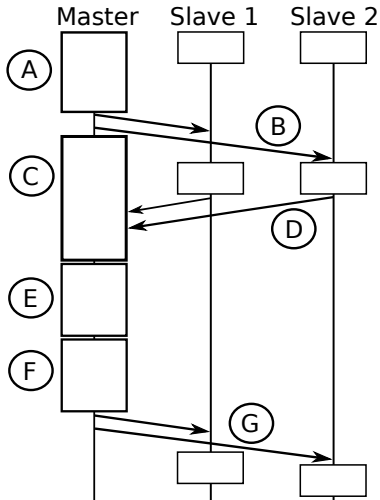
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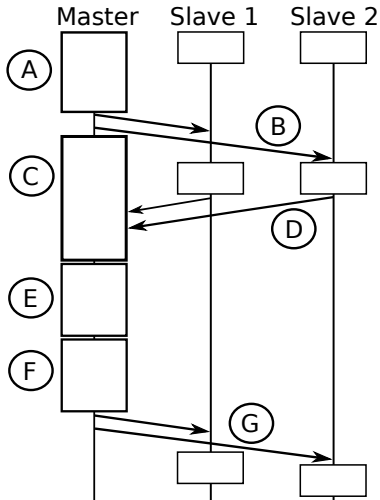
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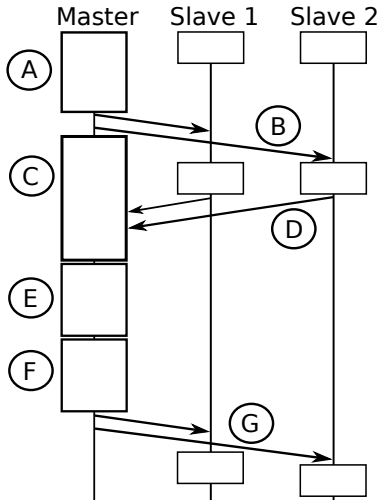


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$$\text{WCET} = \Sigma \text{ A to G}$$

# Analysis of MPI\_Allreduce

- WCET of sequential parts estimated with OTAWA
- Worst-case traversal time (WCTT) of communication parts has to be added
- Result: Equation with parameters
  - #values to be transmitted
  - #communication partners
  - Dimensions of NoC
  - Transportation times
  - Time between Core and NoC
- Equation can be reused for *any* application on same architecture



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# The CG Benchmark

- Conjugate Gradient method from mathematics
- Optimization method to find the minimum/maximum of a multidimensional function
  - Operations on a large matrix
  - Distributed on several cores
  - Cores exchange data a number of times
- Taken from NAS Parallel Benchmark Suite for highly parallel systems
- Adapted for C + MPI

## Setting for the analysis

- Simple ARM cores with 5-stage pipeline
- Distributed memory, no caches, 10 cycles memory access latency
- 4x4 PaterNoster NoC, arranged as unidirectional torus
- Sending and receiving takes 3 assembler instructions + 4 cycles from Pipeline to NoC
- Due to the usage of time division multiplexing (TDM), a WCTT can be estimated
  - Stegmeier et al.: WCTT bounds for MPI Primitives in the PaterNoster NoC, *14th International Workshop on Real-Time Networks (RTN)*, July 5th, 2016, Toulouse

# Analysis of the CG Benchmark

- Initialization not analysed
- Structure of one benchmark iteration:
  - Alternating sequential and communication parts
  - Some of them are repeated for 15 times in a for loop
- Summation of parts gives equation
  - Shortcoming: ignored pipeline states at summation

$$\begin{aligned}WCET_{cg} &= 1\,896\,959 \\ &+ WCET_{AR}(2, 15) + 17 \cdot WCET_{AR}(1, 3) \\ &+ 16 \cdot (WCET_{AR}(351, 3) + WCET_{SR}(351))\end{aligned}$$

# Results

Specific numbers:

- $WCET_{AR}(2, 15) = 8.158 \text{ cycles}$
- $WCET_{AR}(1, 3) = 1.071 \text{ cycles}$
- $WCET_{AR}(351, 3) = 113.073 \text{ cycles}$
- $WCET_{SR}(351) = 11.396 \text{ cycles}$

$$\begin{aligned} WCET_{cg} &= 1\,896\,959 \\ &+ 8\,158 + 17 \cdot 1\,071 \\ &+ 16 \cdot (113\,073 + 11\,396) \\ &= 3\,914\,828 \text{ cycles} \end{aligned}$$



# Outline

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- Learn from parallel programming models
- MPI collectives: clear separation of computation and communication phases
  - ⇒ combine separate WCET estimates for sequential code and MPI collectives
- Analyzed in case study: MPI\_Allreduce, MPI\_Sendrecv, CG benchmark
- Results can be reused for any application on same architecture



# Summary and Outlook

- Learn from parallel programming models
- MPI collectives: clear separation of computation and communication phases
  - ⇒ combine separate WCET estimates for sequential code and MPI collectives
- Analyzed in case study: MPI\_Allreduce, MPI\_Sendrecv, CG benchmark
- Results can be reused for any application on same architecture

## Outlook

- Improve MPI implementation: e. g. workload distribution
- Optimize hardware support



Questions?

Thank you for your attention!



# Additional slides

## Equation of MPI\_Allreduce

- $f$ : #values to be transmitted
- $\chi$ : #communication partners
- $n$ : Dimensions of NoC
- $t_{transm,\chi}$ : Transportation times
- $t_{Buf}$ : Time from Core to NoC and backwards

$$\begin{aligned} WCET_{AR}(f, \chi) = & 273 + 35f\chi + 141\chi \\ & + \max(23 + 6n^2 + 11\chi, 24 + 2(t_{transm,\chi} + t_{Buf})) \\ & + (f - 1)\max(35\chi, t_{transm,\chi}) \\ & + (66 + t_{transm,\chi})f + t_{Buf} \end{aligned}$$

## Equation of MPI\_Sendrecv

- $f$ : #values to be transmitted
- $t_{transm,x}$ : Transportation times
- $t_{Buf}$ : Time from Core to NoC and backwards

$$WCET_{SR}(f) = 108 + 2 \cdot (t_{transm,1} + t_{Buf}) + \max(f \cdot 32, t_{transm,f}) + t_{Buf}$$